Chapter 3: PROBABILITY

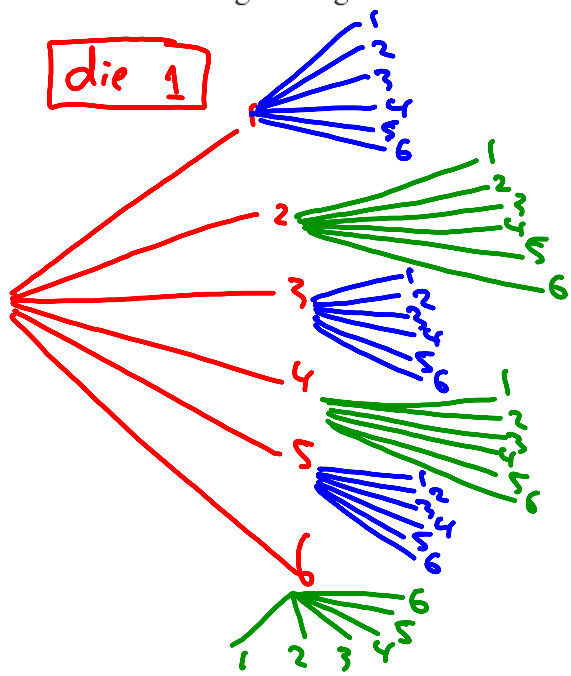
3.1 Exploring Probability:

$$P(event) = \frac{number\ of\ outcomes\ favourable\ to\ the\ event}{total\ number\ of\ outcomes\ in\ the\ sample\ space}$$

counting from unit 2

An **event** is any collection of possible outcomes of an experiment, that is, any subset of the sample space.

Investigation: Draw a tree diagram to show all the possible outcomes of rolling two regular six-sided dice:



36 out comes

1. What is the total number of possible outcomes when rolling two six-sided dice?

36 out comes

2. How many outcomes have double numbers?

double 1's, 2's ...., 6's 6 outcomes

3. What is the probability of rolling two different numbers?

$$P(\text{Rolling 2 different \#'s}) = \frac{\text{\# of rolls of 2 different \#'s}}{\text{Total \# of rolls}}$$

$$= \frac{30}{36} \div 6 = \frac{5}{6} \checkmark$$

or

$$= 0.833$$

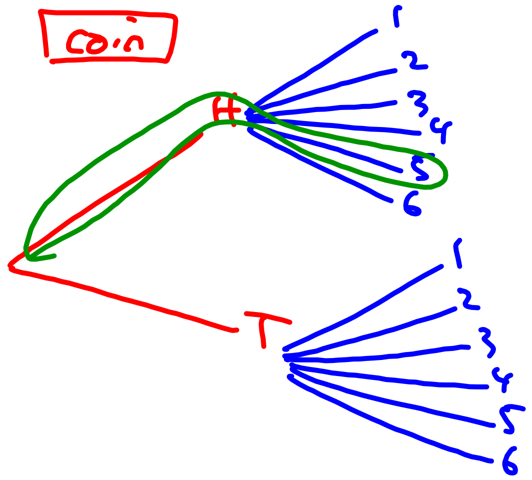
or

$$= 83.3\%$$

Example 1

A) List the sample space of an experiment in which one coin is tossed and one six-sided die is rolled at the same time.

Create a tree diagram



2×6

$= 12$ out comes

B) Determine the probability of the event of tossing a head and rolling a 5.

$P(1H \text{ AND one } 5) = \frac{1}{12}$

or

$= \underbrace{P(1H)} \times \underbrace{P(\text{one } 5)}$

$= \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

C) Determine the probability of the event of tossing a head and rolling either a 4 or a 2.

$P(H \text{ and } (4 \text{ or } 2)) = \frac{2}{12} \div 2$

$= \frac{1}{6}$

or

$\underbrace{P(H)} \times P(4 \text{ or } 2)$

$= \frac{1}{2} \times \frac{2}{6} = \frac{2}{12} = \frac{1}{6}$

A game is considered fair when all the players are equally likely to win. 100%

0%

- ⇨ The probability of an event can range from 0 (impossible) to 1 (certain).
- ⇨ You can express probability as a fraction, a decimal, or a percent.

<u>3.2 PROBABILITY AND ODDS</u>

What is the difference between <u>probability</u> and <u>odds?</u>

- There is a 60% chance of rain this evening.

- The odds of rolling a 3 on a dice is 1:5

<u>PROBABILITY:</u>

- ➢ **Probability** of a particular event compares the ratio of the number of ways the event can occur to the number of possible outcomes.
- ➢ The probability of an event may be written as a fraction, decimal, or percent.
- ➢ When outcomes have an equal chance of occurring, they are **equally likely**.
- ➢ When an outcome is chosen without any preference, the outcome occurs at **random**.

$P(A) = \frac{n(A)}{n(S)}$,where $n(A)$ is the number of times event A occurs and $n(S)$ is the total number of possible outcomes.

<u>ODDS:</u>

The **odds** of an event occurring is the ratio of the number of ways the event can occur (favorable) to the number of ways the event cannot occur (unfavorable).

Odds = favorable : unfavorable

Example 1
Out of 25 people, 10 are teens.
The odds that a person is a teen would be 10 (favorable) to 15 (unfavorable).

10:15

reduce since both ÷5

2:3

The **odds in favour** is the ratio of favourable outcomes to unfavourable outcomes.

Odds in Favour = n(A) : n(A')
[fav : unfav]

The **odds against** is the ratio of unfavourable outcomes to favourable outcomes

Odds Against = n(A') : n(A)
[unfav : fav]

*Hint: The odds against are the reciprocal of the odds in favour!

Note: The odds are always expressed as a ratio in lowest terms

Examples:

1. Find the odds of randomly selecting the letter *p* in the word "Mississippi."

2 p's

odds in fav FAV : UNFAVOURABLE

2 : 9

2. What are the odds of randomly selecting a dime from a dish containing 11 pennies, 6 nickels, 5 dimes, and 3 quarters?

unf unf FAV unf

FAV : UNF

$5 : 20 \xrightarrow{\div 5} \boxed{1 : 4}$

3. A) Eleven poker chips are numbered consecutively 1 through 10, with two of them labeled with a 6 and placed in a jar. A chip is drawn at random. Find the probability of drawing a 6.

P (Getting 6) = $\frac{\text{\# of 6's}}{\text{\# of chips}}$ = $\frac{2}{11}$ or 0.182

or 18.2%

B) What are the odds in favor of the event happening?

2 : 9

C) What are the odds against the event happening?

9 : 2

4. Suppose that the odds in favor of an event are 5:3. What is the probability that the event will happen?

$P = \frac{5}{8}$ ← (5+3) ← Total

5. A computer randomly selects a university student's name from the university database to award a $100 gift certificate for the bookstore. The odds against the selected student being male are 57:43. Determine the probability that the randomly selected university student will be male.

odds against
57:43
UNF FAV(MALE)

P(male) = FAV / Total = 43 / 100 ← 57 + 43

6. A hockey game has ended in a tie after a 5 min overtime period, so the winner will be decided by a shootout. The coach must decide whether Ellen or Brittany should go first in the shootout. The coach would prefer to use her best scorer first, so she will base her decision on the player's shootout records.

Player	Attempts	Goals Scored
Ellen	13	8
Brittany	17	10

} check probability %

Who should go first? P(E) = 8/13 = 0.615 or 61.5%

P(B) = 10/17 = 0.588 or 58.8%

Ellen should go first

7. A group of grade 12 students are holding a charity carnival to support a local food bank. The students have created a dice game that they call Bim and a card game that they call Zap. The odds against winning Bim are 5:2 and the odds against winning Zap are 7:3. Which game should Nolan play?

P(win Bim) = 2/7 = 0.286 → 28.6%

P(win ZAP) = 3/10 = 0.3 → 30%

Nolan should play ZAP

P(A′) is the probability of the complement of A, (not A) where **P(A′) = 1 – P(A)**

100% – % of A occurring

Example: If the probability of winning is 70%, the complementary event (opposite) would be the probability of losing, which is

100% – 70%
= 30%

Questions: pg. 148-150 #1,2, 4-12,14

3.3 PROBABILITIES USING COUNTING METHODS

Example 1: A lock has a three-digit code. Determine the probability that the lock code will consist of three different odd digits.

$$P(\text{three odd digits}) = \frac{n(favorable)}{n(total\ outcomes)}$$

Example 2: Two cards are picked **without replacement** from a deck of 52 playing cards. Determine the probability that both are kings.

Is order important?

$$P(K_1 \text{ and } K_2) = \frac{n(favorable)}{n(total\ outcomes)}$$

$$P(K_1 \text{ and } K_2) = \frac{n(select\ 2K\ out\ of\ 4)}{n(select\ 2cards\ out\ of\ 52)}$$

Example 3: The athletic council decides to form a sub-committee of seven council members to look at how funds raised should be spent on sports activities at the school. There are a total of 15 athletic council members, 9 males and 6 females. What is the probability that the sub-committee will consists of exactly 3 females?

$$P(3\text{F}) = \frac{n(exactly\ 3F)}{n(select\ sub{-}committee\ of\ 7)}$$

Example 4: A bag of marbles contains 5 red, 3 green, and 6 blue marbles. If a child grabs three marbles from the bag, determine the probability that:

A) exactly 2 are blue:

Example 4: A bag of marbles contains 5 red, 3 green, and 6 blue marbles. If a child grabs three marbles from the bag, determine the probability that:

B) at least one is blue:

C) the first is red, the second is green and the third is blue:

Example 4: A bag of marbles contains 5 red, 3 green, and 6 blue marbles. If a child grabs three marbles from the bag, determine the probability that:

D) One is red, one is green and one is blue:

Questions: pg. 159-160 #3,4,6a.8,10

Example 5: If a 4-digit number is generated at random from the digits 2, 3, 5 and 7 (without repetition of the digit), what is the probability that it will be even?

Example 6: Mike, Evan and Allan are competing with 7 other boys to be on the cross-country team. All boys have an equal chance of winning the trial race. Determine the probability that Mike, Evan and Allan will place first second and third

Example 7: To win a prize at a local radio station, a contestant needs to spell out the word SASKATOON with letter tiles. If the tiles are mixed up and all face down, what is the probability that the contestant will win the prize?

Questions: pg. 159-162 #1,2,5,11,12,14,16

Mid-Chapter Review Questions: pg. 165 #3-9

3.4 MUTUALLY EXCLUSIVE EVENTS

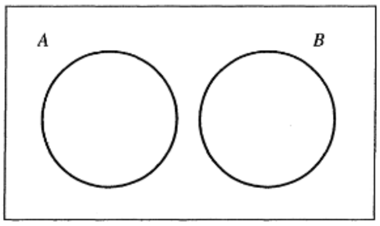
- Events are said to be mutually exclusive if they have **no common outcomes**. (no overlap or intersection)
- For mutually exclusive events we are looking for the word "OR"

The probability formula for **mutually exclusive** events:

P(A or B) = P(A) + P(B)

Investigation:
Consider the experiment of drawing a card from a regular 52 card deck.
Let the event A be "a heart is drawn" and event B be "a spade is drawn."

1. Mark the outcomes to the experiment on the Venn Diagram which represents the sample space. Because these events have _____ common outcomes, we say the events A and B are ____________ ________________.



2. Determine the P(A or B)

P (A or B) = P(A) + P(B)

Examples

1. State whether the events A and B are mutually exclusive or not.

(A) Experiment – a card is drawn from a standard deck

Event A – a face card is selected Event B – a diamond is selected

Ask: Do the two events have anything in __________________?

(B) Experiment – two dice are thrown
Event A – the dice both show the same value Event B – the total score is 8

(C) Experiment – two dice are thrown
Event A – the dice both show the same value Event B – the total score is 9

2. A single die is rolled. What is the probability of rolling a 2 or a 6?

3. A single card is drawn from a standard deck of cards. What is the probability of drawing a red card or a black queen?

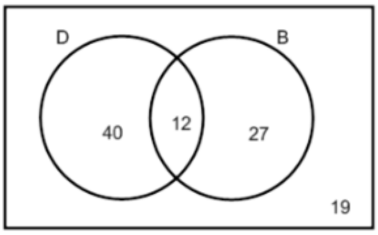
When Events are NOT Mutually Excusive

- Events have an intersecting set (overlap)

The probability formula for **non-mutually exclusive** events:

$$\boxed{\mathbf{P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)}}$$

Example 1: In the Venn diagram, D represents students on the debate team, B represents students on the basketball team.



(A) Are the events mutually exclusive?

(B) What is the probability that students are on the debate team **or** the basketball team?

Example 2: In a class survey,
63% play sports
27% play a musical instrument
20% play neither a sport or musical instrument.
Are these events mutually exclusive?

Example 3: A school newspaper published the results of a recent survey of students' eating habits. 62% said they skip breakfast, 24% skip lunch, and 22% eat both breakfast and lunch.

(A) Are skipping breakfast and lunch mutually exclusive events?

(B) Determine the probability that a randomly selected student skips breakfast only.

(C) Determine the probability that a randomly selected student skips breakfast or lunch.

Example 4: A car manufacturer keeps a database of all the cars that are available for sale to all the dealerships in Western Canada. For model A, the database reports that 43% have heated seats, 36% have a sunroof, and 49% have neither. Determine the probability of a model A car at a dealership having both heated seats and a sunroof.

Questions: pg. 176-179
#3-5,7-9,13-15

3.5 CONDITIONAL PROBABILITY

⇨ ***Conditional Probability:*** the probability of an event occurring given that another event has occurred.

Investigate
A computer manufacturer knows that, in a box of 100 chips, 3 will be defective.
Jocelyn will draw 2 chips, at random, from a box of 100 chips.
What is the probability that both the chips will be defective?

1. Draw a Tree Diagram to represent the ways you can draw two computer chips from the box.

2. Name the four permutations for the situation.

3. Which of the 4 permutations are we concerned with?

4. What is the probability of drawing a defective chip on the first pull?

5. What is the probability of drawing a defective chip on the second pull? Does the first draw impact the probability of the second pull? If so, how?

6. How can I determine the probability of drawing two defective chips?

7. Suppose that Jocelyn replaced the first chip before drawing the second chip. Would the probability of the second chip being defective remain the same?

8. Explain why the probability of drawing a defective chip on the second draw is considered a conditional probability.

9. Go back to the tree diagram drawn on step 1. Label each branch with its probability. Determine the probability of drawing each permutation of defective and not defective chips, then add these probabilities. What does the sum imply?

Dependent Events: Events whose outcomes are affected by each other.

Conditional Probability: The probability of an event occurring given that another event has already occurred.

- If event B depends on event A occurring, then the **conditional probability** that event B will occur, given that event A, has occurred, can be represented as follows:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

- If event B depends on event A occurring, then the probability that both events will occur can be represented as follows:

$$P(A \cap B) = P(A) \cdot P(B|A)$$

Example 1: **Classify** the following as dependent or independent events.

A) The experiment is rolling a die and tossing a coin. The first event is rolling 3 on the die and the second event is tossing heads on the coin.

B) The experiment is drawing two cards without replacement from a standard deck. The first event is drawing a queen and the second event is drawing a queen.

C) The experiment is drawing two cards with replacement from a standard deck. The first event is drawing a jack and the second event is drawing a jack.

Example 2: Cards are drawn from a standard deck of 52 cards without replacement. Calculate the probability of obtaining:

(A) a club then a heart

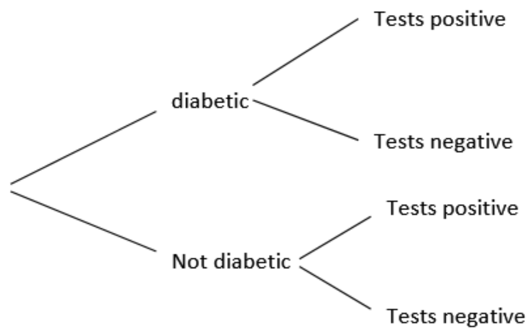
(B) a black card, then a heart, then a diamond

Example 3: According to a survey, 91% of Canadians own a cellphone. Of these people, 42% have a smartphone. Determine, to the nearest percent, the probability that any Canadian you met during the month in which the survey was conducted would own a cell phone that is a smartphone.

Example 4. A test for Type 2 diabetes (non-insulin dependent) measures the blood glucose level after eight hours of fasting. Consider a blood glucose level above normal to be a positive result and anything else to be a negative result. This test is 85% accurate, and 2% of the world's population actually has diabetes.

Determine:

(A) A tree diagram for the situation



(B) the probability that an individual tests positive for diabetes.

Example 5: Andrea likes to go for a daily run. If the weather is nice she is 85% likely to run 5km. If the weather is rainy she is on 35% likely to run 5km. The weather forecast for tomorrow indicates a 60% chance of rain. Determine the probability that Andrea will jog for 5km.

Questions: pg. 188-191
#1-10,13,16,18,19

3.6 INDEPENDENT EVENTS

Investigate: Consider the following situations.

Situation 1	Situation 2
Situation 1 **Drawing two cards from a deck** *without* replacement **A: The first card is a spade** **B: The second is a spade** How many cards were in the deck on the first draw? How many of these cards were spades?	**Situation 2** **Drawing two cards from a deck** *with* replacement **A: The first card is a spade** **B: The second is a spade** How many cards were in the deck on the first draw? How many of these cards were spades?
How many cards were in the deck on the second draw? How many of these cards were spades?	How many cards were in the deck on the second draw? How many of these cards were spades?
Calculate P(A and B) ***These are dependent events***	Calculate P(A and B) ***These are independent events***

Dependent Events: Two events are dependent if after the first event has occurred, it effects the probability of the other event occurring. The probability of event B depends on whether or not event A occurred.

Independent Events: Two events are independent if after the first event has occurred it has no effect on the probability of the second event occurring. The probability of event B does not depend on whether or not event A occurred.

To determine the probability of two events occurring we can use the formula:

P(A and B) = P(A) x P(B)

Example 1: What is the probability of rolling a 3 on a die and tossing heads on a coin?

Example 2: Jane encounters 2 traffic lights on her way to school. There is a 55% chance that she will encounter a red light at the first light, and a 40% chance that she will encounter a red light on the second light. If the traffic lights operate on separate timers, determine the probability that both lights will be red on her way to school,

Example 3: The probability that Ashley will pass Math this semester is 0.7 and the probability that she will pass English this semester is 0.9. If these events are independent, determine the following to the nearest hundredth:

A) Ashley will pass math and English

B) Ashley will pass math but not English

C) Ashley will pass English but not Math

D) Ashley will pass neither

Questions: pg. 198-200
#1,5,6,8,9,13