Lesson 5 Solving Problems Using Quadratic Function Models

Goal: **Solving problems involving situations that can be modeled by quadratic functions.**

Remember:	**Forms of a Quadratic Function**	
(i)	**Standard Form**	$y = ax^2 + bx + c$
(ii)	**Factored Form**	$y = a(x - r)(x - s)$
(iii)	**Vertex Form**	$y = a(x - p)^2 + q$

(I) Applying quadratic models in problem solving.

Example:
A quarterback throws the ball from an initial height of 6 feet. It is caught by the receiver 50 feet away, at a height of 6 feet. The ball reaches a maximum height of 20 feet during its flight. Determine the quadratic function which models this situation and state the domain and range.

- **Sketch a picture**

Based on the information given in the problem, select the best form for modeling the problem.

- **If the vertex is given select Vertex Form**
- **If x-intercepts are given select Factored From**

- **Substitute the given information into the quadratic form that was selected and determine the value of 'a'.**

- **Once the value of 'a' is determined, write the quadratic form that models the problem.**

(II) Solving a maximum/minimum problem with a quadratic function in standard form.

Example:
A boat in distress fires off a flare. The height of the flare, *h*, in metres above the water, *t* seconds after shooting, is modeled by the function $h(t) = -4.9t^2 + 29.4t + 3$. Algebraically determine the maximum height attained by the flare.

(III) Representing a situation with a quadratic model.

(A) Revenue Problems

Example:
A travel agency offers a group rate of $2400 per person for a week in London if 16 people sign up for the tour. For each additional person who signs up, the price per person is reduced by $100. How many people, in total, must sign up for the tour in order for the travel agency to maximize their revenue? Determine the maximum revenue.

Formula for revenue:

Revenue = (number sold)(cost)

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Lesson 5 Solving Area Problems Using Quadratic Function Models (Part II)

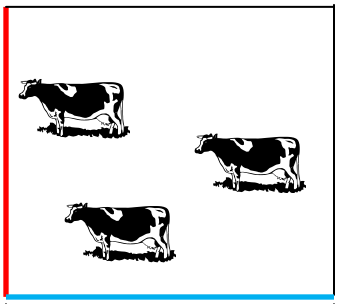
(I) Open field example:

Example 1

A farmer is constructing a rectangular fence in an open field to contain cows. There is 120 m of fencing. Write the quadratic function that models the rectangular region, and use it to determine the maximum area of the enclosed region.

Formula for Area of a rectangle:

Area = (width)(length)



Width =

Length =

Step I

•which variable represents width _____

Step II

•develop the expression for length as a function of width.

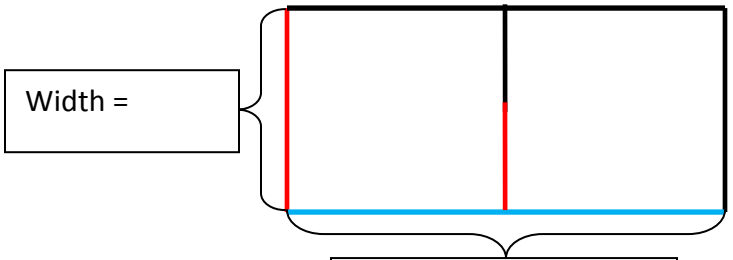
•NOTE: when constructing a fence in an open field, _____ of the fencing is used when the first width and length sides are constructed.

Step III

•Using the width and length expressions, develop the area function in standard form.

Example 2

You have 600 meters of fencing and a large field. You want to make a rectangular enclosure split into two equal lots. Write the quadratic function that models the rectangular region Use the function to determine the dimensions would yield an enclosure with the largest area?

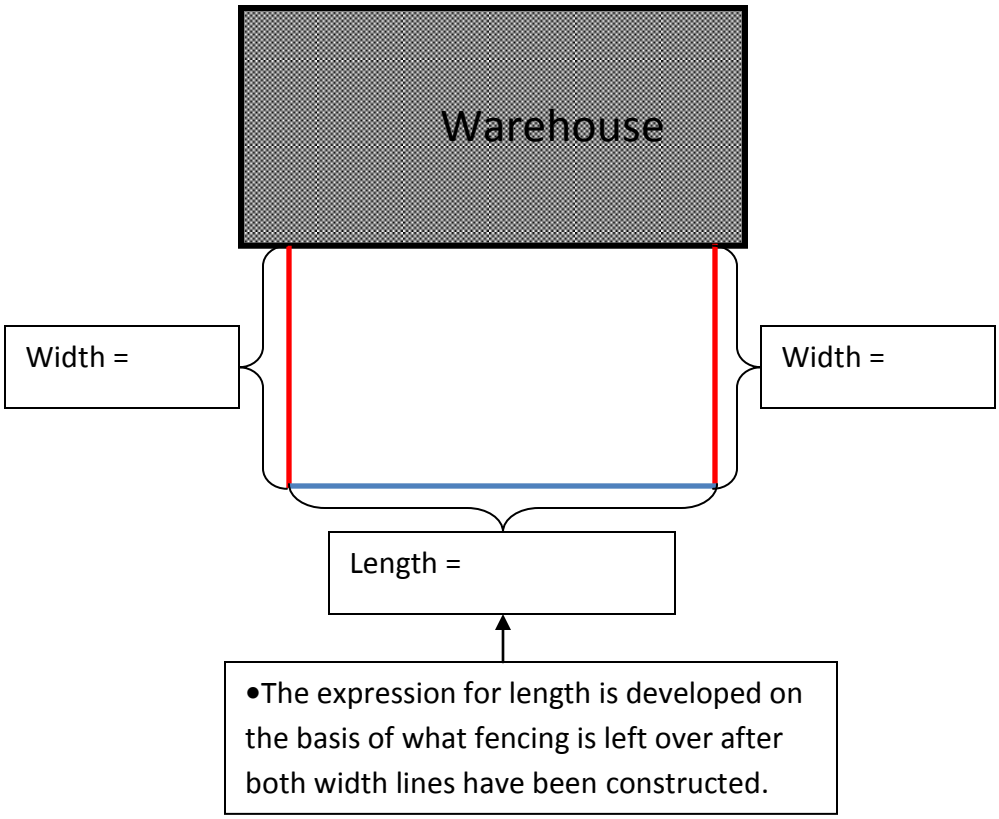


When constructing a fence in an open field, _____ of the fencing is used when half of the width and length sides are constructed.

(II) Using a physical structure as one side

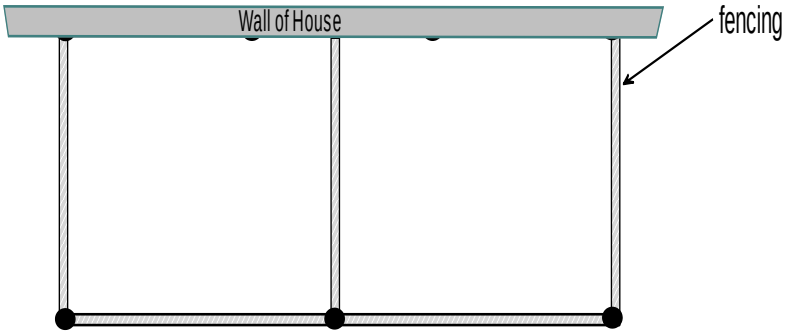
Example 1

A Heavy Equipment Operator has 200 m of fencing to construct a rectangular storage area using a Warehouse as one side. Write the quadratic function that models the rectangular region, and use it to determine the maximum area of the enclosed region.



Example 2

A rectangular region, placed against the wall of a house, is divided into two regions of equal area using a total of 150 m of fencing as shown.



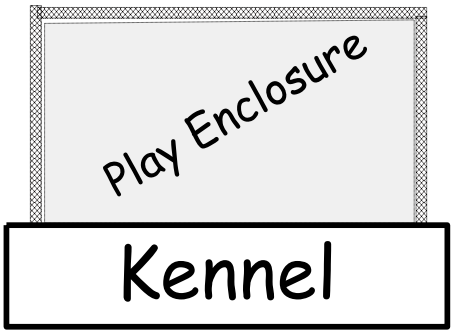
(a) Develop a quadratic function that models the area of the pen.

(b) Determine the maximum area of the pen.

(c) State the domain and range.

Area Practice Problems

1. A rectangular play enclosure for some dogs is to be made with 40 m of fencing using the kennel as one side of the enclosure as shown.

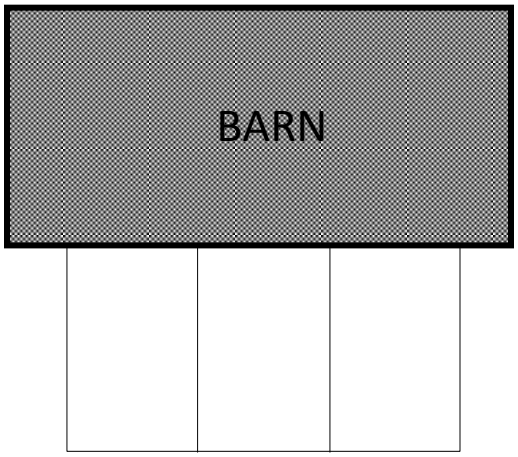


   (a) Develop a quadratic function that models the area of the pen.

   (b) Determine the maximum area.

   (c) State the domain and range of the variables in the function.

2. A barn which contains different livestock will use 240 m of fencing to construct three equal rectangular regions. There is no fencing along the side of the barn so livestock can move in and out of the barn.



   (a) Develop a quadratic function that models the area of the pen.

   (b) Determine the maximum area of the pen and state the dimensions.

3. A lifeguard marks off a rectangular swimming area at a beach with 200 m of rope using the beach as one side. Determine the maximum area and the dimensions of the swimming area?

4. A farmer is going to construct a rectangular fence in an open field using 400m of fencing. Develop an appropriate quadratic function and use it to determine the maximum enclosed area and the dimensions of the rectangular region.

5. A rectangular storage area for heavy equipment is to be constructed using 148 m of fencing and a building as one side. Set up an appropriate equation and use it to determine the dimensions required to maximize the area enclosed.

Answers: 1(a) $A = -2x^2 + 40x$ (b) 200 m^2 (c) domain $0 < x < 20$ range $0 < y \leq 200$
2.(a) $A = -4x^2 + 240x$ (b) 3600 m^2 30m x 120 m
3. 5000 m^2 50m x 100 m 4. $A = -x^2 + 200x$ 10 000 m^2 100m x 100 m
5. 37m x 74 m